Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Monday 13 May 2024

Afternoon (Time: 1 hour 40 minutes) | **Paper reference** | **8FM0/01**

Further Mathematics

Advanced Subsidiary

PAPER 1: Core Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

1. The cubic equation

$b=-3 \quad d=7$

$$2x^3 - 3x^2 + 5x + 7 = 0$$

$a=2 \quad c=5$

has roots α, β and γ.

Without solving the equation, determine the exact value of

(i) $\alpha^2 + \beta^2 + \gamma^2$ **(3)**

(ii) $\dfrac{3}{\alpha} + \dfrac{3}{\beta} + \dfrac{3}{\gamma}$ **(3)**

(iii) $(5 - \alpha)(5 - \beta)(5 - \gamma)$ **(3)**

$\alpha + \beta + \gamma = -\frac{b}{a} = \frac{3}{2}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{5}{2}$ (1)

$\alpha\beta\gamma = -\frac{d}{a} = -\frac{7}{2}$

(i) considering $(\alpha+\beta+\gamma)^2$

$(\alpha+\beta+\gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2\alpha\beta + 2\beta\gamma + 2\gamma\alpha$

$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$= \left(\frac{3}{2}\right)^2 - 2\left(\frac{5}{2}\right)$ (1)

$= -\frac{11}{4}$ (1)

(ii) $\frac{3}{\alpha} + \frac{3}{\beta} + \frac{3}{\gamma} = \frac{3\beta\gamma + 3\alpha\gamma + 3\alpha\beta}{\alpha\beta\gamma} = \frac{3(\alpha\beta + \beta\gamma + \gamma\alpha)}{\alpha\beta\gamma}$

$= \frac{3(5/2)}{-7/2}$ (1) (1)

$= -\frac{15}{7}$ (1)

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Question 1 continued

(iii) $(5-\alpha)(5-\beta)(5-\gamma)$ expand triple brackets

$= (5-\alpha)(25-5\beta-5\gamma+\beta\gamma)$

$= 125 - 25\beta - 25\gamma + 5\beta\gamma - 25\alpha + 5\alpha\beta + 5\gamma\alpha - \alpha\beta\gamma$

$= 125 + 5(\alpha\beta+\beta\gamma+\gamma\alpha) - 25(\alpha+\beta+\gamma) - \alpha\beta\gamma$ (1)

$= 125 + 5\left(\frac{5}{2}\right) - 25\left(\frac{3}{2}\right) + \frac{7}{2}$ (1)

$= \frac{207}{2}$ (1)

Question 1 continued

Question 1 continued

(Total for Question 1 is 9 marks)

2. With respect to the **right-hand rule**, a rotation through $\theta°$ anticlockwise about the z-axis is represented by the matrix

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Given that the matrix **M**, where

$$\mathbf{M} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

represents a rotation through $\alpha°$ anticlockwise about the z-axis with respect to the **right-hand rule**,

(a) determine the value of α.

(1)

(b) Hence determine the smallest possible positive integer value of k for which $\mathbf{M}^k = \mathbf{I}$

(2)

The 3×3 matrix **N** represents a reflection in the plane with equation $y = 0$

(c) Write down the matrix **N**.

(1)

The point A has coordinates $(-2, 4, 3)$

The point B is the image of the point A under the transformation represented by matrix **M** followed by the transformation represented by matrix **N**.

(d) Show that the coordinates of B are $(2 + \sqrt{3}, 2\sqrt{3} - 1, 3)$

(2)

Given that O is the origin,

(e) show that, to 3 significant figures, the size of angle AOB is 66.9°

(2)

(f) Hence determine the area of triangle AOB, giving your answer to 3 significant figures.

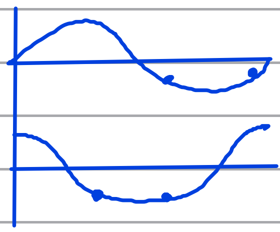
(2)

a) $\cos\alpha = -\frac{\sqrt{3}}{2}$ AND $\sin\alpha = -\frac{1}{2}$

$\alpha = 150, \underline{210}$ $\alpha = \underline{210}, 330$

$\therefore \alpha = 210$ (1)

must be common to both



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Question 2 continued

b) k rotations of $210°$ should be a multiple of 360 for $M^k = I$ (1)

$210 = 2 \times 3 \times 5 \times 7$
$360 = 2^3 \times 3^2 \times 5$

so require $2^2 \times 3 = 12$ to reach a multiple of 360
$k = 12$ (1)

c) $N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (1)

d) M then N $\Rightarrow$ matrix NM

(1)

$$NM = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -\sqrt{3}/2 & 1/2 & 0 \\ -1/2 & -\sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -\sqrt{3}/2 & 1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$NMA = \begin{pmatrix} -\sqrt{3}/2 & 1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} \sqrt{3}+2 \\ 2\sqrt{3}-1 \\ 3 \end{pmatrix}$$ (1)

e) $A = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$ $B = \begin{pmatrix} \sqrt{3}+2 \\ 2\sqrt{3}-1 \\ 3 \end{pmatrix}$ $\cos AOB = \dfrac{\underline{a} \cdot \underline{b}}{|\underline{a}||\underline{b}|}$

$\begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} \sqrt{3}+2 \\ 2\sqrt{3}-1 \\ 3 \end{pmatrix} = 1 + 6\sqrt{3}$ (1)

$\cos AOB = \dfrac{1+6\sqrt{3}}{29}$

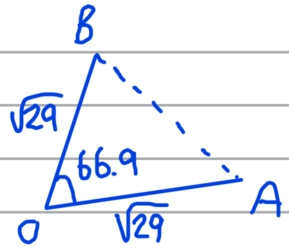
$\sqrt{2^2+4^2+3^2} = \sqrt{29}$

$AOB = 66.9°$ (3sf) (1)

$\sqrt{(\sqrt{3}+2)^2+(2\sqrt{3}-1)^2+3^2} = \sqrt{29}$

Question 2 continued

f) area AOB using $\frac{1}{2}ab\sin C$



Area AOB $= \frac{1}{2}\sqrt{29}\sqrt{29}\sin 66.9°$ (1)

$= 13.3°$ (3sf) (1)

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Question 2 continued

(Total for Question 2 is 10 marks)

3. (a) Use the standard results for summations to show that, for all positive integers n,

$$\sum_{r=1}^{n} r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(an+b)$$

where a and b are integers to be determined.

(4)

(b) Hence show that, for all positive integers k,

$$\sum_{r=k+1}^{3k} r^2(r+1) = \frac{1}{3}k(3k+1)(Ak^2+Bk+C)$$

where A, B and C are integers to be determined.

(3)

(c) Hence, using algebra and making your method clear, determine the value of k for which

$$25\sum_{r=k+1}^{3k} r^2(r+1) = 192k^3(3k+1)$$

(3)

a) $\sum_{r=1}^{n} r^2(r+1) = \sum_{r=1}^{n} r^3 + r^2 = \frac{1}{4}n^2(n+1)^2 + \frac{1}{6}n(n+1)(2n+1)$ 1 1

$= \frac{1}{12}n(n+1)[3n(n+1)+2(2n+1)]$ 1

factor out $\frac{n(n+1)}{12}$

$= \frac{1}{12}n(n+1)[3n^2+7n+2]$

$= \frac{1}{12}n(n+1)(n+2)(3n+1)$ 1

b) $\sum_{r=k+1}^{3k} = \sum_{r=1}^{3k} - \sum_{r=1}^{k} = \frac{1}{12}(3k)(3k+1)(3k+2)(9k+1)$

$- \frac{1}{12}k(k+1)(k+2)(3k+1)$ 1

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Question 3 continued

$= \frac{1}{12}k(3k+1)[3(3k+2)(9k+1) - (k+1)(k+2)]$ 1

$= \frac{1}{12}k(3k+1)[3(27k^2+3k+18k+2) - (k^2+3k+2)]$

$= \frac{1}{12}k(3k+1)[80k^2+60k+4]$

$= \frac{1}{3}k(3k+1)(20k^2+15k+1)$ 1

c) $\frac{25}{3}k(3k+1)(20k^2+15k+1) = 192k^3(3k+1)$

$\frac{25}{3}(20k^2+15k+1) = 192k^2$ ← $\div k(3k+1)$, as $k=0$ and $k=-\frac{1}{3}$ are not valid solutions

$25(20k^2+15k+1) = 576k^2$

$76k^2 - 375k - 25 = 0$ 1

$(76k+5)(k-5) = 0$ 1

$k = 5$ 1

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4.

$$\mathbf{A} = \begin{pmatrix} -1 & -2 & -7 \\ 3 & k & 2 \\ 1 & 1 & 4 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 4k-2 & 1 & 7k-4 \\ -10 & 3 & -19 \\ 3-k & -1 & 6-k \end{pmatrix}$$

where k is a constant.

(a) Determine the value of the constant c for which

$$\mathbf{AB} = (3k + c)\mathbf{I}$$

(2)

(b) Hence determine the value of k for which $\mathbf{A}^{-1}$ does not exist.

(2)

Given that $\mathbf{A}^{-1}$ does exist,

(c) write down $\mathbf{A}^{-1}$ in terms of k.

(1)

(d) Use the answer to part (c) to solve the simultaneous equations

$$\begin{aligned} -x - 2y - 7z &= 10 \\ 3x + ky + 2z &= 3 \\ x + y + 4z &= 1 \end{aligned}$$

giving the values of x, y and z in simplest form in terms of k.

(3)

a) considering top-left entry:

$-1(4k-2) - 2(-10) - 7(3-k) = 3k - c$ 1

$-4k + 2 + 20 - 21 + 7k = 3k + c$

$3k + 1 = 3k + c$

$c = 1$ 1

b) $AB = (3k+1)I$

if $3k + 1 = 0$, 1 $k = -\frac{1}{3}$ 1

P 7 4 0 6 9 A 0 1 4 3 2

Question 4 continued

c) $AB = (3k+1)I$

$A^{-1}AB = (3k+1)A^{-1}I$

$B = (3k+1)A^{-1}$

$A^{-1} = \frac{1}{3k+1}B$

$= \frac{1}{3k+1}\begin{pmatrix} 4k-2 & 1 & 7k-4 \\ -10 & 3 & -19 \\ 3-k & -1 & 6-k \end{pmatrix}$ (1)

d) $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 3 \\ 1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} 10 \\ 3 \\ 1 \end{pmatrix}$

$= \frac{1}{3k+1}\begin{pmatrix} 4k-2 & 1 & 7k-4 \\ -10 & 3 & -19 \\ 3-k & -1 & 6-k \end{pmatrix}\begin{pmatrix} 10 \\ 3 \\ 1 \end{pmatrix}$ (1)

$= \frac{1}{3k+1}\begin{pmatrix} 47k-21 \\ -110 \\ 33-11k \end{pmatrix}$

$\left(\frac{47k-21}{3k+1}, \frac{-110}{3k+1}, \frac{33-11k}{3k+1}\right)$

(1) (1)

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5. Given that on an Argand diagram the locus of points defined by $|z + 5 - 12\mathrm{i}| = 10$ is a circle,

(a) write down,

(i) the coordinates of the centre of this circle,

(ii) the radius of this circle.

(2)

(b) Show, by shading on an Argand diagram, the set of points defined by

$$|z + 5 - 12\mathrm{i}| \leqslant 10$$

(1)

(c) For the set of points defined in part (b), determine the maximum value of $|z|$

(3)

The set of points A is defined by

$$A = \{z : 0 \leqslant \arg(z + 5 - 20\mathrm{i}) \leqslant \pi\} \cap \{z : |z + 5 - 12\mathrm{i}| \leqslant 10\}$$

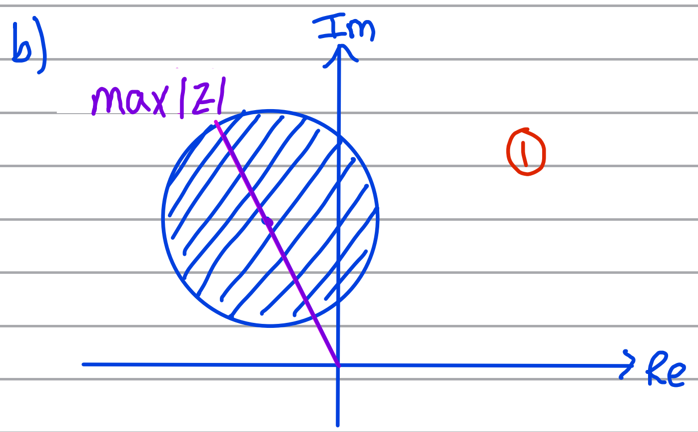
(d) Determine the area of the region defined by A, giving your answer to 3 significant figures.

(4)

a) $|z - (-5 + 12i)| = 10$

centre $(-5, 12)$ 1 , radius 10 1

b)



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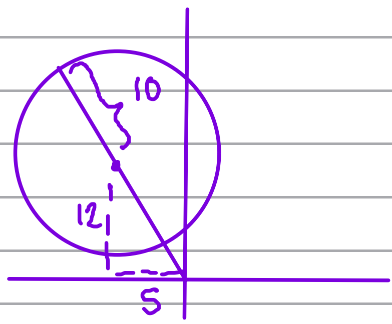
c) Max $|z| = \sqrt{5^2 + 12^2} + 10 = 23$ 1 1 1



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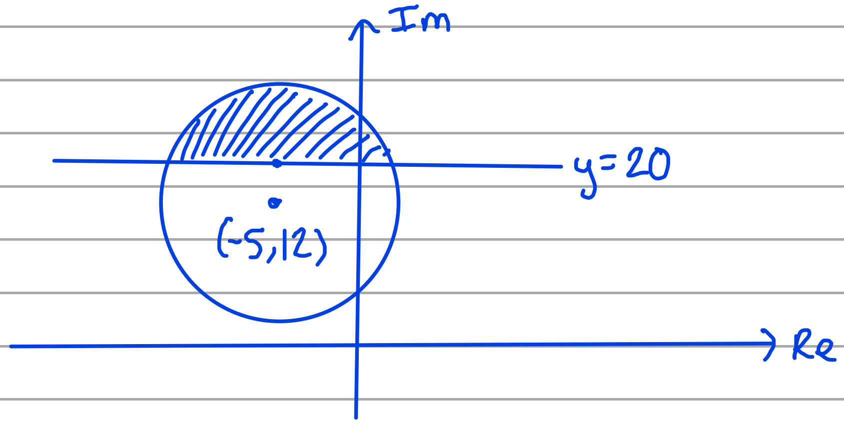
Question 5 continued

d) $0 \le \arg(z - (-5+20i)) \le \pi \Rightarrow y = 20$



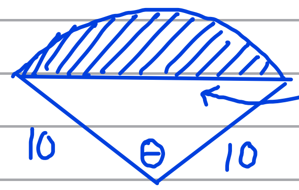
points of intersection of circle $(x+5)^2 + (y-12)^2 = 100$
and $y=20$: sub in $y=20$

$(x+5)^2 + 8^2 = 100$

$(x+5)^2 = 36$

$x+5 = \pm 6$
$x = -1$ or $x = 11$

area of segment = sector area − triangle area



distance between x-coordinates of intersection points $= 12$

cosine rule to find θ:

$\cos\theta = \dfrac{10^2 + 10^2 - 12^2}{2 \times 10 \times 10} = 0.28$ ①

$\theta = \cos^{-1}(0.28)$
$\theta = 1.287...$ rad ①

Question 5 continued

area of segment $= \frac{1}{2} \times 10^2 \times \theta - \frac{1}{2} \times 10 \times 10 \times \sin\theta$ 1

$= 16.4$ (3sf) 1

using $\frac{1}{2}\theta r^2$ using $\frac{1}{2}ab\sin C$

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Question 5 continued

(Total for Question 5 is 10 marks)

6. The drainage system for a sports field consists of underground pipes.

This situation is modelled with respect to a fixed origin O.

According to the model,

- the surface of the sports field is a plane with equation $z = 0$
- the pipes are straight lines
- one of the pipes, P_1, passes through the points $A(3, 4, -2)$ and $B(-2, -8, -3)$
- a different pipe, P_2, has equation $\dfrac{x-1}{2} = \dfrac{y-3}{4} = \dfrac{z+1}{-2}$
- the units are metres

(a) Determine a vector equation of the line representing the pipe P_1

(2)

(b) Determine the coordinates of the point at which the pipe P_1 meets the surface of the playing field, according to the model.

(2)

Determine, according to the model,

(c) the acute angle between pipes P_1 and P_2, giving your answer in degrees to 3 significant figures,

(3)

(d) the shortest distance between pipes P_1 and P_2

(5)

a) direction vector $\overrightarrow{BA} = \begin{pmatrix}3\\4\\-2\end{pmatrix} - \begin{pmatrix}-2\\-8\\-3\end{pmatrix} = \begin{pmatrix}5\\12\\1\end{pmatrix}$ (1)

$\underline{r} = \begin{pmatrix}3\\4\\-2\end{pmatrix} + \lambda\begin{pmatrix}5\\12\\1\end{pmatrix}$ (1)

b) want intersection of $\underline{r}$ with $z=0$

$\Rightarrow -2 + \lambda = 0$ $\leftarrow$ setting z coordinate of $P_1 = 0$

$\lambda = 2$ (1)

$\underline{r} = \begin{pmatrix}3+2(5)\\4+2(12)\\0\end{pmatrix} = \begin{pmatrix}13\\28\\0\end{pmatrix}$

coordinates are $(13, 28, 0)$ (1)

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Question 6 continued

c) Vector equation of P_2: $r = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$

acute angle between two lines:

$\cos\theta = \left| \frac{b_1 \cdot b_2}{|b_1||b_2|} \right|$ where b_1 and b_2 are the direction vectors of the lines

$b_1 = \begin{pmatrix} 5 \\ 12 \\ 1 \end{pmatrix}$ $b_2 = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$

$\begin{pmatrix} 5 \\ 12 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$

$= 10 + 48 - 2 = 56$ (1)

$\sqrt{5^2+12^2+1^2} = \sqrt{170}$ $\sqrt{2^2+4^2+2^2} = \sqrt{24}$

$\cos\theta = \frac{56}{\sqrt{170}\sqrt{24}}$ (1) $\Rightarrow \theta = 28.8^\circ$ (3sf) (1)

d) general point on P_1: $\begin{pmatrix} 3+5\lambda \\ 4+12\lambda \\ -2+\lambda \end{pmatrix}$

general point on P_2: $\begin{pmatrix} 1+2\mu \\ 3+4\mu \\ -1-2\mu \end{pmatrix}$

Method: find general vector between P_1 and P_2, then set it perpendicular to both P_1 and P_2 using dot product

$P_1 - P_2$: $\begin{pmatrix} 3+5\lambda-(1+2\mu) \\ 4+12\lambda-(3+4\mu) \\ -2+\lambda-(-1-2\mu) \end{pmatrix} = \begin{pmatrix} 5\lambda-2\mu+2 \\ 12\lambda-4\mu+1 \\ \lambda+2\mu-1 \end{pmatrix}$ (1)

P 7 4 0 6 9 A 0 2 3 3 2

Question 6 continued

setting perpendicular to P_1:

$$\begin{pmatrix} 5\lambda - 2\mu + 2 \\ 12\lambda - 4\mu + 1 \\ \lambda + 2\mu - 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 12 \\ 1 \end{pmatrix} = 0$$

$$5(5\lambda - 2\mu + 2) + 12(12\lambda - 4\mu + 1) + (\lambda + 2\mu - 1) = 0$$

$$170\lambda - 56\mu = -21 \quad (1)$$

setting perpendicular to P_2:

$$\begin{pmatrix} 5\lambda - 2\mu + 2 \\ 12\lambda - 4\mu + 1 \\ \lambda + 2\mu - 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} = 0 \quad (1)$$

$$2(5\lambda - 2\mu + 2) + 4(12\lambda - 4\mu + 1) + 2(\lambda + 2\mu - 1) = 0$$

$$56\lambda - 24\mu = -10 \quad (2)$$

solve (1) and (2) simultaneously

$$\lambda = 7/118 \qquad \mu = 131/236 \quad (1)$$

so $P_1 - P_2 = \begin{pmatrix} 70/59 \\ -30/59 \\ 10/59 \end{pmatrix}$

(1)

$$|P_1 - P_2| = \sqrt{\left(\frac{70}{59}\right)^2 + \left(\frac{30}{59}\right)^2 + \left(\frac{10}{59}\right)^2} = 1.3\text{m} \ (2\text{sf}) \quad (1)$$

Question 6 continued

(Total for Question 6 is 12 marks)

7. (i) Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$$

(5)

(ii) Prove by induction that, for all positive integers n,

$$f(n) = 3^{2n+4} - 2^{2n}$$

is divisible by 5

(5)

(i) • Base case $n=1$

LHS: $\sum_{r=1}^{1} \frac{1}{r(r+1)} = \frac{1}{1(2)} = \frac{1}{2}$ RHS: $\frac{1}{1+1} = \frac{1}{2}$

LHS = RHS $\therefore$ true for $n=1$ (1)

• Assume true for $n=k$ so

$$\sum_{r=1}^{k} \frac{1}{r(r+1)} = \frac{k}{k+1} \quad (1)$$

• for $n=k+1$:

$$\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \sum_{r=1}^{k} \frac{1}{r(r+1)} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} \; (1) = \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)}$$

$$= \frac{(k+1)^2}{(k+1)(k+2)} = \frac{k+1}{k+1+1} \; (1)$$ so true for $n=k+1$

• If true for $n=k$, it has been shown to be true for $n=k+1$. (1)
Since it is true for $n=1$, it is true for all positive integers n.

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Question 7 continued

(ii) $f(n) = 3^{2n+4} - 2^{2n}$

• Base case $f(1) = 3^{2+4} - 2^{2} = 725 = 145(5)$, divisible by 5.

so true for $n=1$ (1)

• Assume true for $n=k$,

$f(k) = 3^{2k+4} - 2^{2k}$ (1)

• consider $f(k+1)$

$f(k+1) = 3^{2(k+1)+4} - 2^{2(k+1)}$

$= 3^{2k+6} - 2^{2k+2}$

$= 3^2(3^{2k+4}) - 2^2(2^{2k})$

$= 9(3^{2k+4}) - 9(2^{2k}) + 5(2^{2k})$ (1)

$= 9f(k) + 5(2^{2k})$ (1)

which is divisible by 5.

• If true for $n=k$ then it has been shown to be true for $n=k+1$. Since it is true for $n=1$, it is true for all positive integers n. (1)

Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

8.

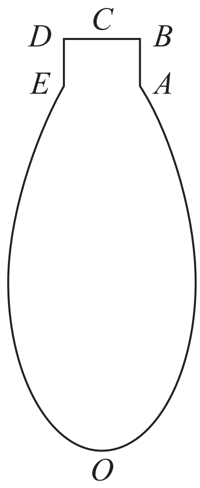


Figure 1

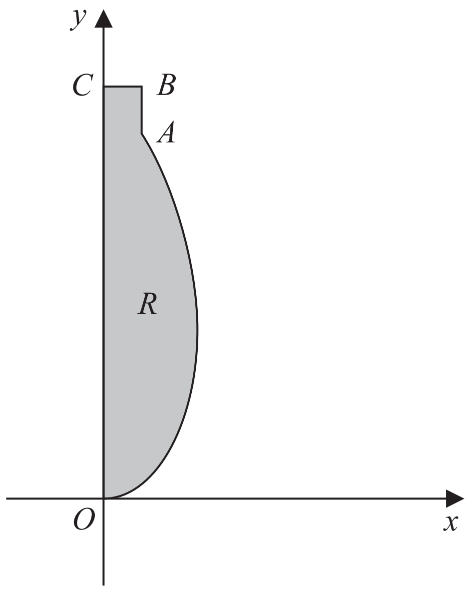


Figure 2

Figure 1 shows the central vertical cross-section, *OABCDEO*, of the design for a solid glass ornament.

Figure 2 shows the finite region, *R*, which is bounded by the *y*-axis, the horizontal line *CB*, the vertical line *BA*, and the curve *AO*.

The ornament is formed by rotating the region *R* through 360° about the *y*-axis.

The curve *AO* is modelled by the equation

$$x = ky^2 + \sqrt{y} \qquad 0 \leqslant y \leqslant 4$$

where k is a constant.

The point *A* has coordinates (0.4, 4) and the point *B* has coordinates (0.4, 4.5)

The units are centimetres.

(a) Determine the value of k according to this model.

(2)

(b) Use algebraic integration to determine the exact volume of glass that would be required to make the ornament, according to the model.

(7)

(c) State a limitation of the model.

(1)

When the ornament was manufactured, $9\,\text{cm}^3$ of glass was required.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning.

(1)

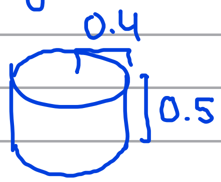
Question 8 continued

a) sub in $x = 0.4$, $y = 4$:

$$0.4 = k \times 4^2 + \sqrt{4} \quad 1$$
$$16k = 0.4 - 2$$
$$k = -0.1 \quad 1$$

b) total volume = cylinder volume + volume of revolution of curve.

cylinder:



$$V = \pi \times 0.4^2 \times 0.5$$
$$= 2\pi/25 \quad 1$$

curve: $x = ky^2 + \sqrt{y}$

$$x^2 = k^2y^4 + 2ky^{5/2} + y \quad 1$$

$$V = \pi \int_0^4 (k^2y^4 + 2ky^{5/2} + y)\,dy \quad 1$$

$$= \pi \left[\frac{k^2}{5}y^5 + \frac{4k}{7}y^{7/2} + \frac{1}{2}y^2 \right]_0^4 \quad 1$$

$k = -0.1$

$$= \pi \left(\frac{256}{125} - \frac{256}{35} + 8 \right) - 0\pi \quad 1$$

$$= \frac{2392}{875}\pi$$

$$\text{total volume} = \frac{2392}{875}\pi + \frac{2}{25}\pi = \frac{2462}{875}\pi \quad 1 \quad 1$$

Question 8 continued

c) the sides of the ornament may not be perfectly smooth (1)

d) it is a good estimate as 8.84cm^3 is only 0.16cm^3 less than 9cm^3 (1)

(Total for Question 8 is 11 marks)

TOTAL FOR CORE PURE MATHEMATICS IS 80 MARKS

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